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Using Differential GPS Positioning for Elevation Determination

DEPARTMENT OF THE ARMY
U.S. Army Corps of Engineers
CECW-EP Washington, DC 20314-1000 ETL 1110-1-183

Technical Letter
No. 1110-1-183 1 April 1998

Engineering and Design
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ELEVATION DETERMINATON

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1. Purpose

This engineer technical letter provides technical guidance for using Differential Global Positioning System (DGPS) to determine elevations of survey benchmarks for wide-area mapping and GIS database development applications. Recommended procedural specifications for field DGPS observation sessions are included.

2. Applicability

This letter is applicable to major subordinate commands, districts, laboratories, and field operating activities having responsibility for civil works, military construction, or environmental restoration projects. These DGPS guidelines and specifications are intended for densifying vertical control over large project areas, such as an entire military installation or watershed basin mapping project. The DGPS methods outlined in this letter are generally ***not*** intended, nor would be cost-effective, for small projects or any type of construction lay out work where vertical grades or benchmarks require an accuracy better than 30 millimeters (mm). In such cases, conventional differential leveling methods should be used.

3. References

a. FGCC, (1991), Standards and Specifications for Geodetic Control Networks, Silver Spring, Maryland. (FGCC is currently known as FGCS)

b. Milbert, D.G. and Smith D.A. (1996). Converting GPS Height into NAVD88 Elevation with the Geoid96 Geoid Height Model. National Geodetic Survey, Silver Spring, Maryland.

c. U.S. Army Corps of Engineers (1994), Deformation Monitoring and Control Surveying. Engineer Manual 1110-1-1004, U.S. Army Corps of Engineers, Washington, D.C.

d. U.S. Army Corps of Engineers (1996), NAVSTAR Global Positioning System Surveying. Engineer Manual No. 1110-1-1003, U.S. Army Corps of Engineers, Washington, D.C.

e. Zilkoski, D.B., D'Onofrio, Joseph D., and Frankes, Stephen J. (1997) Guidelines for Establishing GPS-Derived Ellipsoid Heights (Standards: 2 cm and 5 cm), Version 4.1.1. Silver Spring, Maryland. NGS Unpublished Report.

f. Zilkoski, D.B., Richards, J.H., and Young, G.M. (1992). Special Report: Results of the General Adjustment of the North American Vertical Datum of 1988, Silver Spring, Maryland.

4. Distribution

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5. Discussion

a. Global Positioning System (GPS) surveying produces a set of X-Y-Z coordinates which can be transformed into geodetic latitude, longitude, and ellipsoidal height by using an reference ellipsoid to model the earth. In the U.S., most GPS ellipsoid heights are measured with respect to North American Datum of 1983 (NAD83) control values, which are based on the Geodetic Reference System of 1980 (GRS80) ellipsoid. Published orthometric elevations on national vertical control benchmarks in the North American Vertical Datum of 1988 (NAVD88) height system are established with respect to the geoid, a model of the earth based on gravity measurements. A determination of a NAVD88 elevation using GPS measurements at a given point requires a transformation between ellipsoid and geoid based height systems. The conversion between the NAD83 GPS ellipsoid and NAVD88 orthometric height is made using the geoidal undulation (also referred to as geoid height) value that represents the geoid-ellipsoid separation distance.

b. DGPS may provide an efficient and cost-effective means of densifying elevation data over large, extended project areas when compared to conventional differential leveling. Height measurement accuracy that meets most USACE mapping requirements can be successfully achieved from several different GPS surveying techniques. However, DGPS vertical elevation techniques may not be sufficiently accurate for construction control or may not be cost-effective for small project areas.

c. GPS relative vertical positioning and calculated geoid height differences for the determination of NAVD88 orthometric heights may be used when an accuracy no better than 30 mm is required. This GPS height accuracy satisfies feature elevation tolerances specified for most USACE engineering mapping activities. However, it may not be sufficiently accurate for hydraulic engineering studies or construction activities. Guidance for GPS survey accuracies and procedures can be found in EM 1110-1-1003.

d. Recent advances in geoid modeling have also led to more accurate conversions between NAD 83 GPS and NAVD 88 orthometric height systems. Accuracies of 30 mm or better have been obtained when converting ellipsoid heights from GPS surveys, based on NAD 83 control, to NAVD 88 orthometric heights using the latest geoid model (GEOID96). The initial GPS survey data must be valid for the elevation transfer method to be effective. Guidance for GPS survey accuracies and procedures can be found in EM 1110-1-1003, NAVSTAR Global Positioning System Surveying.

e. Appendix A presents the basic methodology for using GPS to determine NAVD88 elevations. GPS positioning techniques, coordinate systems, and vertical datum concepts are introduced and discussed along with operational requirements and computational schemes used to obtain NAVD88 elevations from GPS coordinates. These operational requirements are based on field test results conducted by U.S. Army Topographic Engineering Center (CETEC) and the National Geodetic Survey (NGS) using several different GPS surveying methods and comparing these results to conventional differential leveling networks.

6. Proponency and Technical Assistance

The HQUSACE proponent for this technical letter is CECW-EP. Technical assistance in performing GPS surveys may be obtained by contacting the U.S. Army Topographic Engineering Center, ATTN: CETEC-TD-G, 7701 Telegraph Road, Alexandria, VA 22315-3864, (703) 428-6767.

FOR THE DIRECTOR OF CIVIL WORKS

Appendix A
Determination of Elevations
with GPS Surveying Techniques

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Appendix A

Determination of Elevations with GPS Surveying Techniques

A-1. Global Positioning System Ellipsoid Heights

a. Recent advances in GPS technology and the current fully operational status for the NAVSTAR GPS have made it possible to accurately measure ellipsoidal height differences from GPS satellites. GPS surveys report vertical positions in geodetic coordinates defined with respect to the World Geodetic System of 1984 (WGS84) reference ellipsoid, a geocentric, bi-axial, ellipsoid of revolution, that is symmetric about the equatorial axis and whose shape and size are defined by mathematical constants corresponding to the length of its equatorial and polar axes and selected to best approximate the geoid (currently within 100 meters globally). Although many different geodetic datums exist throughout the world, WGS84 is the reference system most frequently used with off-the-shelf GPS equipment. The ellipsoidal height value at a given point is based on the distance measured along the normal vector from the surface of the reference ellipsoid to the point. The realization and practical accuracy of WGS84 as a vertical reference frame for collecting elevation data depends on the actual ellipsoidal height values assigned to benchmarks or other physically defined control points.

b. In the U.S., final positions from DGPS are established with respect to NAD83. Since NAD83 is based on the GRS80 ellipsoid, ellipsoid heights obtained from GPS surveying using NAD83 control are based on the GRS80 ellipsoid. These heights are referred to as NAD83 GPS ellipsoidal heights. Unlike the WGS84 ellipsoid, the GRS80 ellipsoid is not exactly geocentric which can create problems (i.e., large errors) when converting NAD83 GPS ellipsoid heights to orthometric heights using some geoid models.

A-2. Orthometric Heights and NAVD88 Elevations

a. The orthometric height of a point is the distance from the reference surface to the point, measured along the line perpendicular to every equipotential surface in between. A series of equipotential surfaces can be used to represent the gravity field. One of these surfaces, the geoid, is specified as the referenced system from which orthometric heights are measured. The geoid itself is defined as a potential surface and natural variations in gravity induce a smooth, continuous, curvature to the plumb line and therefore physical equipotential surfaces which are normal to gravity do not remain geometrically parallel over a given vertical distance (i.e. the plumb line is not quite parallel to the ellipsoidal normal).

b. The NAVD88 datum is the product of a vertical adjustment of leveled height difference measurements made across North America. NAVD88 was constrained by holding fixed the orthometric height of a single primary tidal benchmark at Father’s Point / Rimouski, Quebec, Canada and performing a minimally constrained general adjustment of U.S.-Canadian-Mexican leveling observations. The vertical reference surface is therefore defined by the surface on which the gravity values are equal to the control point value. NAVD88 elevations are published orthometric heights that represent the geometric distance from the geoid to the terrain measured along the plumb line. Orthometric height corrections were used to enforce consistency between geopotential based vertical coordinates and measured leveled differences. NAVD88 is the most compatible vertical reference frame available to relate GPS ellipsoidal heights to orthometric heights. Further information on the NAVD88 datum can be found in Zilkoski (1992) and EM 1110-1-1004, Deformation Monitoring and Control Surveying.

A-3. Geoidal Heights

a. Geoidal heights or (geoid height values) represent the geoid-ellipsoid separation distance measured along the ellipsoid normal and are obtained by taking the difference between ellipsoidal and orthometric height values (see Figure 1). Knowledge of the geoid height enables the evaluation of vertical positions in either the geodetic (ellipsoid based) or the orthometric height system through the following relation:

$$h = H + N$$

where, h = ellipsoid height
H = orthometric height
N = geoid height,

and by convention N being a positive height above the ellipsoid.

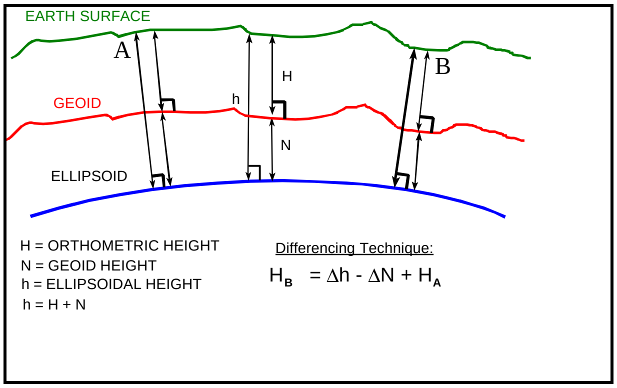

Figure 1. Geoid/Ellipsoid Relationship

b. Geoid height values at stations where either only **h** or **H** is known can be obtained from geoid models which are mathematical surfaces representing the shape of the Earth's gravity field. The geoid model is constructed from a truncated functional series approximation using a spherical harmonics expansion and an extensive set of globally available gravity data. The model is determined from the unique coefficients of the finite series representing the geoid surface. Its accuracy depends on the coverage and accuracy of the gravity measurements used as boundary conditions. Former geoid models produced for general use limit absolute accuracies for geoid heights to no less than 1 meter. More recent geoid models have shown a significant increase in absolute accuracy for geoid heights to a few centimeters.

c. In practice the shape of the geoid surface is estimated globally as a function of horizontal coordinates referenced to a common geocentric position. Specific geoid height values are extracted from the model surface at the node points of a regular grid (i.e., a 2' x 2' grid spacing). Biquadratic interpolation procedures can be used within a grid cell boundary to approximate the geoid height at a given geodetic latitude and longitude. The NGS GEOID96 model for the United States indicates geoid heights range from a low of -51.6 meters in the Atlantic to a high of -7.2 meters in the Rocky Mountains. For more information on geoid modeling, see Milbert (1996).

A-4. Relative Vertical Positioning with GPS

a. DGPS observation sessions produce 3-D geodetic coordinate differences that establish the baseline between two given stations. Baseline solutions produce relative positioning results at a greater accuracy than can now be achieved from point positioning. The expected accuracy of such ellipsoidal height difference measurements is based on several factors such as GPS receiver manufacture type, observation session duration, and the measured baseline distance, but it does not depend greatly on prior knowledge of the absolute vertical position of either occupied station. Dual frequency, carrier phase measurement based GPS surveys are usually able to produce 3-D relative positioning accuracies under 30 mm at the 95% confidence level over baseline distances less than 20 km, depending on the type of GPS surveying method used. This situation exists mainly because GPS range biases are physically well correlated over relatively short distances and tend to cancel out as a result of forming double differences for carrier phase data processing. In contrast, GPS absolute code positioning accuracy will contain the full effects of any GPS range measurement errors. The method explained below to obtain NAVD88 elevations from satellite surveys is based on the relative vertical positioning capability of GPS.

b. Geoidal height differences describe the change in vertical position of the geoid with respect to the ellipsoid between two stations. These relative geoidal heights can be more accurate than the modeled absolute separation values within extended areas because the relative geoidal height accuracy is based on the continuous surface characteristics of the geoid model, where only small deviations between closely spaced points would be expected. The regional trend or slope of the geoid at a given point will not be highly sensitive to local gravity anomalies especially in non-mountainous areas.

A-5. GPS - NAVD88 Transformation Techniques

a. The GPS relative positioning and processing concepts used for NAVD88 elevation determination can be outlined as follows:

(1) GPS baselines are observed to determine ellipsoidal height differences across a network of stations. This step corresponds to a standard GPS network survey using whatever observation techniques are best suited for the project accuracy requirements (i.e. static, rapid static, kinematic, On-The-Fly (OTF), or other geodetic quality GPS surveying process). GPS precise ephemeris should be used in place of the broadcast ephemeris during baseline data processing. Repeat baselines, for all baselines, should be performed for all control surveys established with DGPS. The average ellipsoid height from the repeat observations will be closer to the truth, with a few exceptions, than the ellipsoid height value from a single observation. Baselines should be reobserved on different days with significantly different satellite constellations. For topographic and location surveys (using kinematic or OTF techniques), repeat occupations should be performed where feasible. It is important that the positions be adjusted on NAD83 since most geoid models (GEOID93, GEOID96) are also based on NAD83.

(2) At least two or more established NAVD88 first order benchmarks should be occupied to serve as the GPS reference stations where accurate vertical coordinates will be fixed for the network adjustment. It is suggested that at least one (preferably 2 or more) of these benchmarks are also High Precision Geodetic Network (HPGN)/ High Accuracy Reference Networks (HARN) stations to ensure accurate geoid modeling. First order accuracy standards for geodetic leveling ensure the relative vertical position of these reference monuments will agree. Redundant vertical control within the project area will provide a check on the solution heights of the unknown stations. The ideal condition would be to have all benchmarks with high order vertical and horizontal control surrounding and within the project area. See EM 1110-1-1003, NAVSTAR Global Positioning System Surveying, for assistance in setting up a network.

(3) Geoid heights at the reference stations are determined from a published geoid model such as GEOID93 or GEOID96. The geoid height is added to the published orthometric height at the GPS reference station to determine its ellipsoid height to the accuracy level of the geoid model.

(4) Once the reference stations' ellipsoidal, orthometric and geoidal heights have been fully determined according to the above methods, elevations are transferred from the reference stations to the remaining points in the network according to the following relation:
Given,

$H_i = h_i – N_I$ (where *i* is the station of unknown height)

$H_{ref} = h_{ref} - N_{ref}$, (where *ref* is the station of known heights)

with measured difference in ellipsoid height ($\Delta h = h_i - h_{ref}$) from a DGPS survey, and computed difference in Geoid height ($\Delta N = N_i - N_{ref}$) from a known Geoid model, then,

$$H_i = H_{ref} + (H_i - H_{ref})$$

$$H_i = H_{ref} + (h_i - N_i) - (h_{ref} - N_{ref})$$

$$H_i = H_{ref} + (h_i - h_{ref}) + (N_i - N_{ref})$$

$$H_i = H_{ref} + (\Delta h - \Delta N)$$

where H_i is the orthometric height of the i-th station, the quantity Δh is determined from the measured GPS ellipsoidal height differences, and the quantity ΔN is the geoidal height difference computed from the geoid model (see Figure 1).

b. The expected precision of the orthometric height from using GPS relative positioning and modeled geoid heights and the above relation can be calculated by the summation of variance components corresponding to the accuracy of the published orthometric height, the GPS relative height determination, and the computed geoid height differences.

c. Positional accuracy for orthometric heights on benchmarks must be obtained from published sources based on the results of a vertical network adjustment. Without this information it is presumed that a fixed vertical control point contributes no additional error to the height of the unknown stations. The uncertainties in GPS relative heights are estimated from the vertical component error estimate produced from the GPS data processing and adjustment software. An error estimate of 10 mm is commonly seen as the minimum baseline error produced from static type surveys. Relative geoidal height ($\sigma_{\Delta N}$) precision from geoid modeling can have an expected standard deviation of 10-20 mm.

d. The aforementioned error values lead to an expected uncertainty in final orthometric height at the unknown station of approximately 30 mm (at the 95% confidence level) relative to the published elevation at the benchmark reference station. A repeatable accuracy of 30 mm meets or exceeds most feature elevation tolerances specified for many USACE surveying and mapping projects, excepting certain high precision surveys such as for structural deformation monitoring. In areas with obstructions, dense vegetation, or high relief between monuments or projects site, GPS can exceed leveling accuracy when time is critical to the project.

A-6. Results of Field Testing

a. Based on an evaluation of DGPS data and geoid modeling software capabilities by CETEC, it was determined that higher accuracy elevations are obtained by the transfer of ellipsoidal height differences and relative geoidal heights from a station with a known NAVD88 elevation, than is possible from the direct application of absolute geoid heights to GPS networks.

b. This analysis was based on various methods used for determining NAVD88 elevations from GPS ellipsoidal height data. These methods were tested on a network of points having known first-order leveled orthometric heights that were tied to first-order vertical control. Results of the testing indicated that GPS-based surveys could determine NAVD88 elevations to an accuracy of 30 mm when relative heights and differences in geoid heights are applied.

c. Note that the accuracy of NAVD88 elevations determined from DGPS-derived heights and geoid modeling is dependent on the accuracy of the GPS coordinate solution and the geoid model.

A-7. Additional Guidelines and Recommendations

In addition to the guidelines discussed in section A-5, the following procedures and methods are recommended and should be implemented when using GPS for elevation determination:

a. Keep project areas within a 20-kilometer radius of control points. GPS relative positioning accuracy depends in part on the length of the measured baseline. Positioning errors grow in direct proportion to baseline length at a rate of approximately 1 part per million. For networks with an area less than 20 km, the distance dependent error in the GPS vertical component (relative ellipsoid height) will be limited. Occupation times of less than 1 hour (i.e., 20-30 min) should produce good results for these shorter baselines. For project areas greater than 20 km, the occupation times should be increased to a minimum of 2 hours for primary and secondary control points. Control points should be spaced throughout (surrounding and within) the project area.

b. Observe when VDOP is less than 5.0. Vertical Dilution of Precision (VDOP) is a measure of vertical positioning accuracy (due mainly to satellite geometry) relative to the precision of the measurements used to determine the position. Large VDOP values represent poor satellite geometry that will generally produce weak positioning solutions, although VDOP is primarily intended as a quality indicator for GPS code solutions.

c. Use fixed tripods poles. Fixed height tripods provide a consistent station occupation method that can reduce the likelihood of antenna height measurement blunders.

d. Use dual frequency receivers. Dual frequency receivers can correct GPS measurements for ionosphere based range errors. This will extend the feasible baseline length and resolve integer ambiguities reliably within 20 km. Dual frequency receiver should be used on all baselines longer than 10 km.

e. Use identical geodetic quality antennas with ground plane. Different makes and models of GPS antennas can have different phase centers. Mixing of different type of antennas can cause errors in the vertical component up to 100 mm. Only if the processing software can account for the phase center difference in the GPS antennas should mixing of antenna types occur. The ground plane on the antenna will reduce the amount of ground reflecting multipath.

f. Occupy points a minimum of twice with different satellite constellations and on different days. The purpose is to ensure different atmospheric conditions (different days) and significantly different satellite geometry (different times) for the two occupations. For example, if the first day observation were made between 8:00 am to 8:30 am, the second 30-minute observation would be made on the next day anytime between 11:30 am and 5:30 pm. If the second observation is not made for a couple of days or even a week, be sure to compensate for the daily 4-minute change in the GPS satellite constellation is accounted for. It has been shown that the average ellipsoid height of repeat observations is closer to the truth, with a few exceptions, than the ellipsoid height of a single observation.

g. Process with a minimum elevation mask of 15 degrees. A 15-degree elevation mask will reduce noise embedded in low elevation satellite data and also minimize potential multipath effects from nearby objects surrounding the antenna. For obstructions low on the horizon, a 20-degree elevation mask may be used during baseline processing.

h. Process GPS data with Precise Ephemeris. The broadcast ephemeris is the prediction of where the satellites will be, but the precise ephemeris is the actual true orbit of the satellites. Use of a precise ephemeris will reduce the error between predicted and actual satellite orbit and increase the accuracy of the survey. The precise ephemeris is available approximately 7 days after a survey through the U.S. Coast Guard Navigation Center home page (www.navcen.uscg.mil) or the National Geodetic Survey home page (www.ngs.noaa.gov).

i. Use only ionosphere free fixed baseline solutions for baselines greater than 10 k. Ionosphere free solutions indicate the use of dual frequency receivers and processing can model and eliminate errors due to signal delay in the ionosphere. Fixed baseline solutions indicate a statistically accurate integer ambiguity was established from the GPS data. A normal, (not ionosphere free) fixed baseline is sufficient for baselines less than 10 km.

j. Use relative geoid height values. Application of the geoid model to both reference and remote stations will produce two absolute geoid heights. The relative geoid height value is determined from the difference between the absolute geoid model height values taken at both ends of a given baseline. Relative geoid heights, when added to measure ellipsoid height differences, produced the best vertical accuracy based on the ground truth test results.

k. For further information on using DGPS for elevations, see Zilkoski (1997).